Dynamic Pricing for Non-fungible Resources

Designing Multi-dimensional Blockchain Fee Markets

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Based on work by T. Diamandis, A. Evans, T. Chitra, G. Angeris

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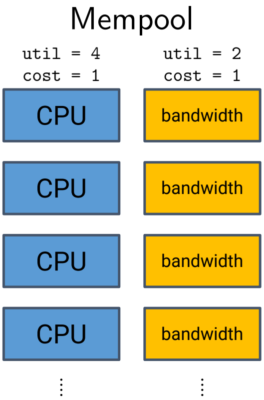
Our work: a framework to optimally set multi-dimensional fees

Outline

Fixed relative prices lead to DoS attacks

- All opcodes have fixed **relative prices** to each other (measured in gas)
- Potential mismatch between relative prices & resource usage leads to **resource exhaustion attacks** (DoS attacks)
  - EXTCODESIZE attack in 2016 exploited disk read mispricing
  - Opcode prices had to be manually adjusted (EIP-150)

Fixed relative prices limit throughput



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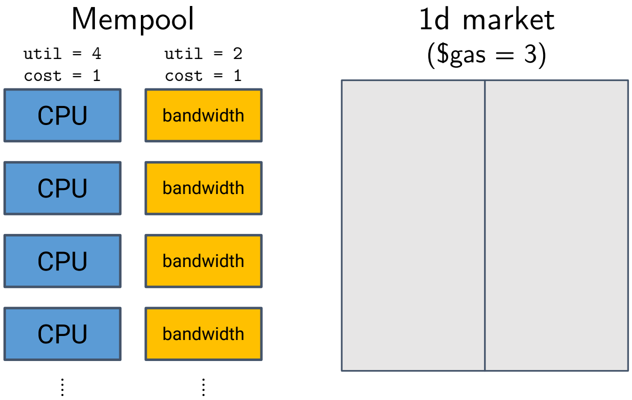

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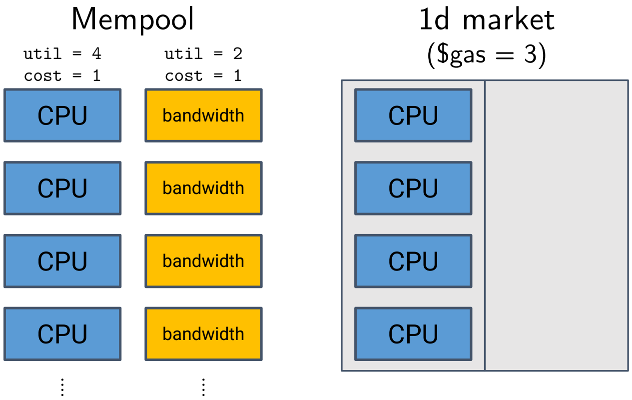

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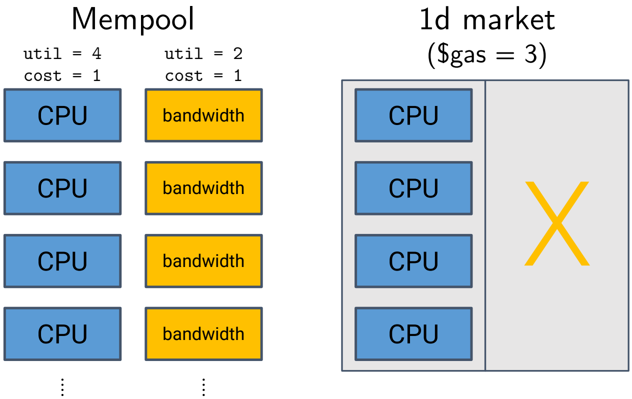

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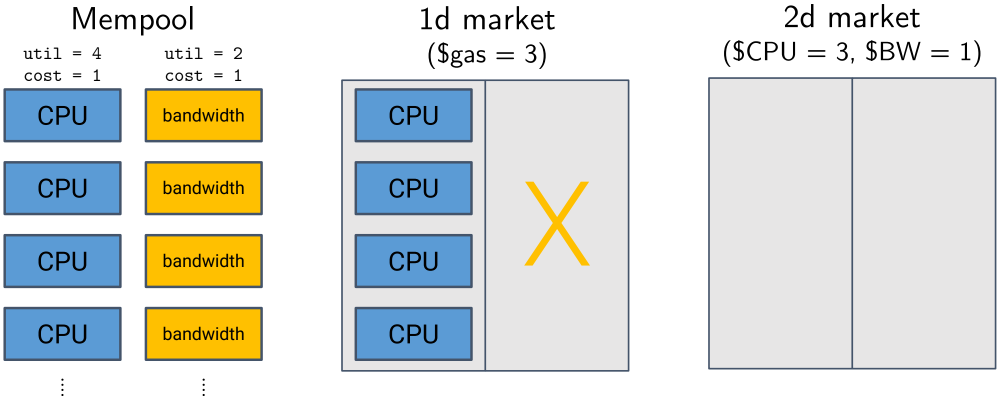

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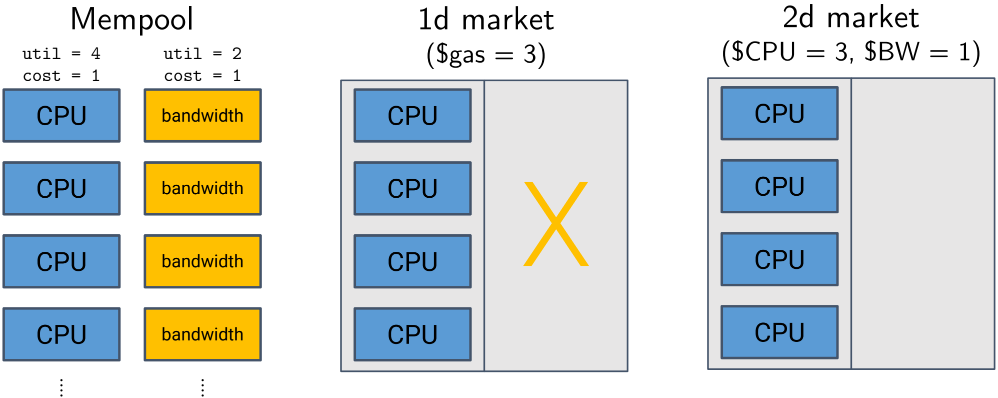

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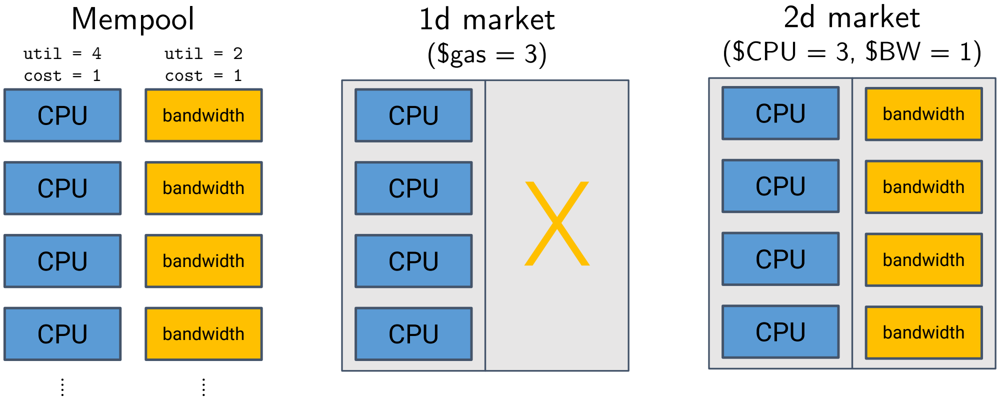

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We need a mechanism to design **fee markets**

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  - Entry $x_j = 1$ if tx j is included and 0 otherwise
- The quantity of resources consumed by this block is then

$$y = \sum_{j=1}^{n} x_j a_j = Ax$$

We constrain & charge for each resource used

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- Charge for usage of each resource (*e.g.*, EIP-1559)
  - Prices p, mean that transaction j costs (this is burned)

$$p^T a_j = \sum_{i=1}^{m} p_i (a_j)_i$$

But how do we determine prices?

- We want a few properties:
  - $(Ax)_i = b_i^\star \to$ no update
  - $(Ax)_i > b_i^\star \to p_i$ increases
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Is this a good update rule?

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Specific choice of objective by network designer $\implies$ specific update rule

Outline

Setting (for now):

Network designer is omniscient and determines txns in each block

Loss function is network's unhappiness with resource usage

- Network designer determines **loss function** for resource allocation problem; *e.g.*:

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We encode all tx constraints in set S

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- We consider the convex hull of S: $\mathrm{conv}(S)$
  - This means j can be 'partially included'
  - $x_j \in (0,1) \implies$ tx j included after roughly $1/x_j$ blocks

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- We almost never know q in practice
- But we will see that the network does not need to know q!

The resource allocation problem

$$\begin{array}{ll} \text{maximize} & q^T x - \ell(y) \\ \text{subject to} & y = Ax \\ & x \in \mathrm{conv}(S). \end{array}$$

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- **Objective:** Maximize utility of included txns minus the loss incurred by the network
- **Constraints:** Utilization y is resource usage of included txns, and x is in the set of allowable txns $S \subseteq \{0, 1\}^n$ (can be very complex/hard to solve!)

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- But network designer cannot solve this in practice!
  - Doesn't decide which txns are in a block (block builders do this)
  - Doesn't know utilities q
  - Cannot include fractional txns ($x_i \in (0, 1)$)

Outline

Duality theory: relaxing constraints to penalties

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- We will 'decouple' utilization of network and that of tx producers
- Correctly set penalty $\rightarrow$ dual problem $=$ original problem & utilizations are equal

Dual decouples tx produces and network

- Problem is separable, so $g(p)$ decomposes into two easily interpretable terms:

$$g(p) = \underbrace{\ell^*(p)}_{\text{network}} + \underbrace{\sup_{x \in \mathrm{conv}(S)} (q - A^T p)^T x}_{\text{tx producers}}$$

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- Evaluating the 1st term is easy: can be done on chain! Let's look at the 2nd term

Second term: block building problem

- Maximize net utility (utility minus cost) subject to tx constraints

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- Same optimal value if we use S instead of $\mathrm{conv}(S)$
- Exact problem solved by block producers! $\rightarrow$ Network can observe $x^\star$

What do we get at optimality?

- Let $p^\star$ be a minimizer of $g(p)$, *i.e.*, prices are set optimally
- Assume the block building problem has optimal solution $x^\star$
- The optimality conditions are

$$\nabla g(p^\star) = y^\star - Ax^\star = 0$$

where $y^\star$ satisfies $\nabla \ell(y^\star) = p^\star$

Key results

1. Prices that minimize g charge the tx producers exactly the marginal costs faced by the network:

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2. These prices incentivize tx producers to include txns that maximize welfare generated $q^T x$ minus the network loss $\ell(Ax)$

Cool. So how do we minimize $g(p)$?

- We can compute the gradient:

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- Network determines $y^\star(p)$ (computationally easy)
- Network observes $x^\star(p)$ from previous block (block building problem soln)
- Then network applies favorite optimization method (*e.g.*, gradient descent)

$$p^{k+1} = p^k - \eta \nabla g(p^k)$$

Some simple examples:

Update rule

$$p^{k+1} = p^k - \eta(b^\star - Ax^\star)$$

Loss function

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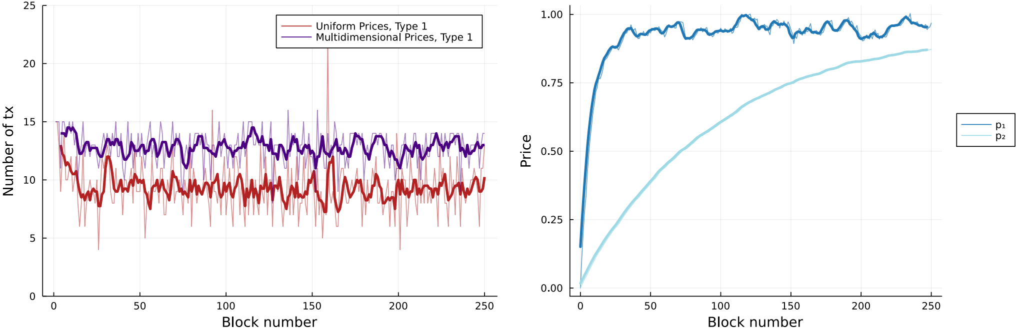
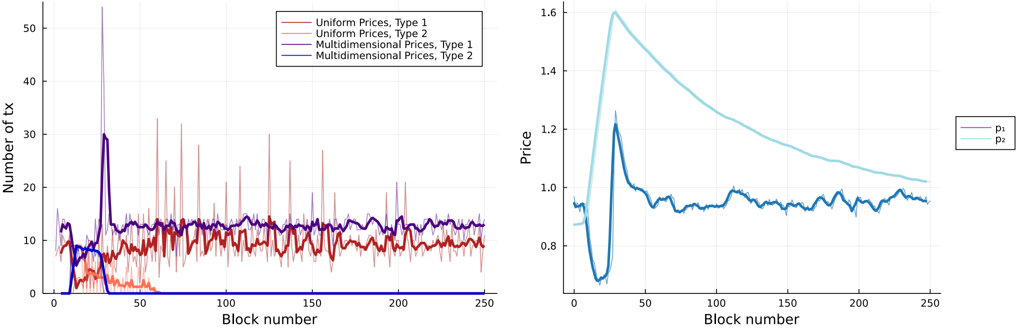
$p_i^{k+1} = p_i^k \cdot \exp\left(\eta(Ax - b^\star)_i\right)$	See paper, appendix C

Outline

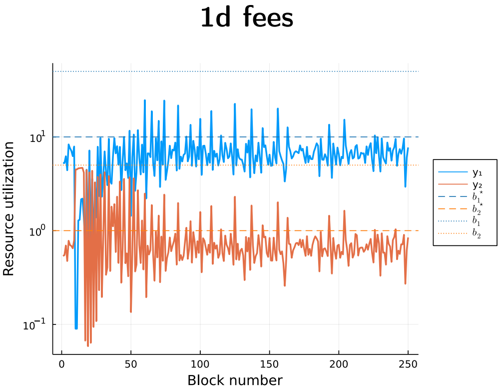
Multidimensional fees increase throughput



Even when the tx distribution shifts



And resource utilitaztion better tracks targets



Conclusion: choose your objective, not the update rule!

Choice of **objective function** by network designer yields an "optimal" price update rule via our optimization-based framework

For more info, check out our paper!

Paper

Thank you!

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