Unfairness in 'Fair' Ordering

Quantifying the Welfare Gap in FIFO Mechanisms

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Based on work by T. Diamandis, G. Angeris

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Caveat: this talk is about problems, not solutions.

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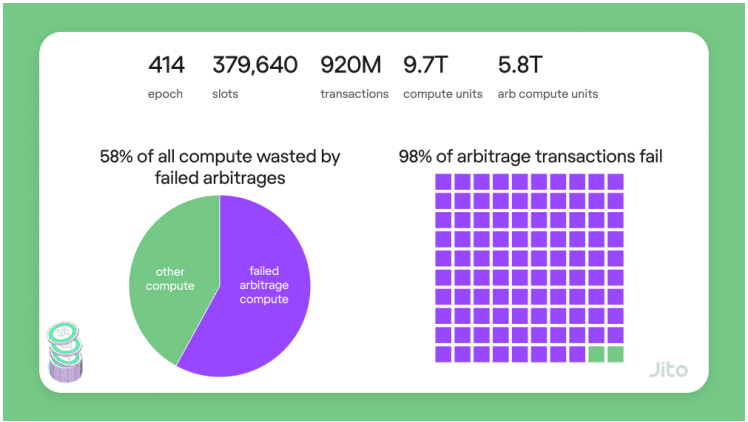
- Solana (w/o Jito) effectively uses FIFO-ordering today (and close to negligible fees)
- What happens if there is a competitive arbitrage opportunity?
- Searchers spam the network trying to be the 'first' transaction!
- This creates huge externalities, borne by the network and its other users.

And the end result is very bad for network performance



Credit: Jito Foundation (Feb 28, 2023)

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- And what's the proposal now? A priority fee! (`TimeBoost`; Ed's talk later today)

We need a market to allocate block space.

Transactions and resources

- A **transaction** j consumes an amount of gas $a_j \in \mathbf{R}_+$
  - Gas has a per-unit cost $g \in \mathbf{R}_+$
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  - Gas has a per-unit cost $g \in \mathbf{R}_+$
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- Each transaction j has utility (net of gas cost) $q_j \in \mathbf{R}_+$
- A vector $x \in \{0,1\}^n$ records which of n possible txns are included in a block
  - Entry $x_j = 1$ if tx j is included and 0 otherwise
  - The **block building problem** is to choose a utility-maximizing x

The block building problem

- Maximize net utility (utility minus cost) subject to tx constraints

$$\begin{array}{ll} \text{maximize} & q^T x \\ \text{subject to} & a^T x \leq b \\ & x \in \{0,1\}^n \end{array}$$

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- **Optimal value** denoted by $p^\star$
- Of course, this is a simplification! More constraints in reality.
- But this problem captures enough for our purposes

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- We think of transactions in terms of utility per unit gas ('efficiency'): q_i/a_i
- We sort the transactions from high to low efficiency, with indices τ_i
- If no txn gas cost is too large, greedy heuristic is close to optimal:

$$a_i \leq b/m \qquad \implies \qquad p^{\mathrm{greedy}} \leq p^\star \leq \frac{m}{m-1} p^{\mathrm{greedy}}$$

where m is some integer.

The greedy block

- The greedy block simply takes the highest efficiency transactions until the gas limit is reached.
- The utility of this block is

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- We'll define the average 'high utility' in terms of this block:

$$q^{+} = \frac{1}{\bar{k}} \sum_{i=1}^{\bar{k}} q_{\tau_i},$$

- Lower bound on optimal value: $p^{\text{greedy}} \geq (b/B^{+})q^{+}$, where $a_i \leq B^{+}$.

The FIFO block

- Assume x is ordered by arrival time
  - Assume this is technically possible, ignore Condorcet paradox
- The utility from the FIFO block is the utility from all the transactions we can fit:

$$p^{\mathrm{FIFO}} = \max\{q^T(1_k, 0) \mid a^T(1_k, 0) \leq b,\ k = 0, 1, \ldots, n\}.$$

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- The **welfare gap** is the difference between optimal and FIFO blocks: $p^\star - p^{\mathrm{FIFO}}$
- Bound this using lower bound on p^{greedy} (and $p^\star$) and upper bound on p^{FIFO}.

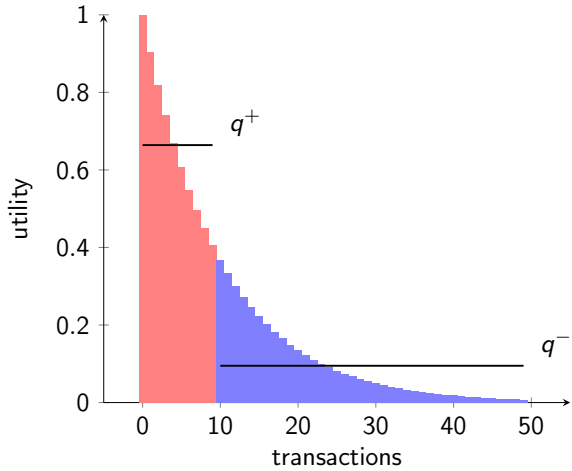
What's the gap?

- Recall, we sort transactions from high to low efficiency: q_i/a_i, with indices τ_i
- If the greedy heuristic block size is $\bar{k}$ transactions, we define the average 'high utility' and 'low utility' by q^+ and q^-, respectively:

$$q^+ = \frac{1}{\bar{k}} \sum_{i=1}^{\bar{k}} q_{\tau_i}, \qquad q^- = \frac{1}{n-\bar{k}} \sum_{i=\bar{k}+1}^{n} q_{\tau_i}.$$

- Intuitively, q^+ is the average utility of the top of the efficiency distribution and q^- is the average utility of the rest.
- We saw that q^+ is the average utility of the greedy block txns.

Often there is a large difference between q^+ and q^-



What's the gap?

- We bound the gas size of the transaction by $B^- \leq a_i \leq B^+$
- Then, we find that the gap is bounded below by

$$p^{\text{FIFO}} - p^\star \geq \frac{b}{B^+} q^+ - \frac{b}{B^-} \left((q^+ - q^-) \frac{\bar{k}}{n} + q^- \right).$$

- This is positive whenever...

$$q^+ \left(1 - \frac{\bar{k}\eta}{n} \right) > \eta q^- \left(1 - \frac{\bar{k}}{n} \right), \tag{1}$$

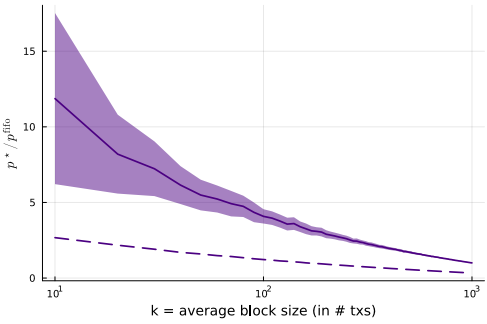
  where $\eta = B^+/B^- \geq 1$

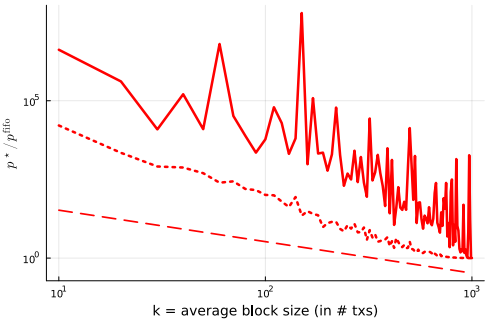
- In practice, frequently have a small number of txns with much higher utility per unit gas (*e.g.*, liquidations).

A simpler bound

- If the number of outstanding transactions is large ($\bar{k}/n$ is small), the gap is positive whenever
$$q^+ > (B^+/B^-)q^-$$
- Roughly, any distribution that isn't flat will lead to a positive welfare gap
- In practice, we see that this bound is quite loose

The gap is large, especially with heavy-tailed distributions





Conclusion: ‘ordering’ transactions causes a welfare gap

FIFO transaction ordering forces benign users to pay for externalities.

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  - And on implementations: SUAVE (Flashbots team), TimeBoost (Arbitrum team, Mamageishvili et al. 2023)
- Others today are talking about these works! (Tarun, Mallesh, Ed)

For more info, check out our short note!

Paper

Thank you!

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